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Discussion

Comments on "The boundary point method for the calculation of exterior acoustic radiation problem" [S.Y. Zhang, X.Z. Chen, *Journal of Sound and Vibration* 228(4) (1999) 761–772]

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Abstract

Zhang and Chen [The boundary point method for the calculation of exterior acoustic radiation problem, *Journal of Sound and Vibration* 228 (1999) 761–772] proposed a boundary point method (BPM) for exterior acoustic problems. The idea is similar to the CHUNKY CHIEF by Wu [A weighted residual formulation for the CHIEF method in acoustic, *Journal of Acoustical Society of America* 90 (1991) 1608–1614], but Chunky CHIEF provides constraints using null-field equations while the BPM used the CHUNKY BLOCK singularity outside the domain. The mathematical structure is similar to Trefftz method and method of fundamental solutions [J.T. Chen et al., On the equivalence of the Trefftz method and method of fundamental solutions for Laplace and biharmonic equations, *Computers & Mathematics with Applications* 53 (2007) 851–879], since the interpolation function satisfies the governing equation. Later, Wu commented twice [Sean F. Wu, Comments on "The boundary point method for the calculation of exterior acoustic radiation" (by S.Y. Zhang, X.Z. Chen, *Journal of Sound and Vibration* 228(4) (1999) 761–772), *Journal of Sound and Vibration*, 298 (2006) 1173]; Sean F. Wu, Comments on "Reply to the comments on 'The boundary point method for the calculation of exterior acoustic radiation' (by S.Y. Zhang, X.Z. Chen, *Journal of Sound and Vibration* 228(4) (1999) 761–772)", *Journal of Sound and Vibration*, 298 (2006) 1176–1177] that the formulation of BPM is wrong and the authors replied also twice [X.Z. Chen, C.X. Bi, Reply to the comments on "The boundary point method for the calculation of exterior acoustic radiation" (by S.Y. Zhang, X.Z. Chen, *Journal of Sound and Vibration* 228(4) (1999) 761–772), *Journal of Sound and Vibration*, 298 (2006) 1174–1175; [X.Z. Chen, C.X. Bi, Reply to the comments on "Reply to the comments on 'The boundary point method for the calculation of exterior acoustic radiation' (by S.Y. Zhang, X.Z. Chen, *Journal of Sound and Vibration* 228(4) (1999) 761–772)", *Journal of Sound and Vibration*, 298 (2006) 1178–1179] to defend themselves. Here, we would like to say a few words for the discussions in two aspects. One is the formulation, and the other is the occurrence of fictitious frequencies.

(1) *Formulation*: Based on the classical Helmholtz integral equation, the linear algebraic system is obtained as

$$[A][\Phi] = [B]\left\{\frac{\partial \Phi}{\partial n}\right\}, \tag{1}$$

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where Φ and $\partial\Phi/\partial n$ are boundary potential and its normal derivative, respectively. We have

$$\left\{\frac{\partial\Phi}{\partial n}\right\} = [B]^{-1}[A]\{\Phi\} \tag{2}$$

and the stiffness matrix is

$$[K] = [B]^{-1}[A]. \tag{3}$$

Based on the method of fundamental solutions or BPM, we have

$$\Phi^*_T(p_i, q_j)c_j = \Phi(p_i), \tag{4}$$

$$\frac{\partial\Phi^*_T(p_i, q_j)}{\partial n_p}c_j = \frac{\partial\Phi(p_i)}{\partial n}, \tag{5}$$

where $\Phi^*_T(p_i, q_j)$ is the fundamental solution for the response at p_i due to a fabricated source at q_j [1–7] and c_j is the strength of the Chunky block. The matrix forms of Eqs. (4) and (5) are expressed as

$$\left[\Phi^*_T\right]\{c\} = \{\Phi\}, \tag{6}$$

$$\left[\frac{\partial\Phi^*_T}{\partial n_p}\right]\{c\} = \left\{\frac{\partial\Phi}{\partial n}\right\}. \tag{7}$$

From Eq. (6), we can determine the unknown strength

$$\{c\} = [\Phi^*_T]^{-1}\{\Phi\}. \tag{8}$$

Substituting Eq. (8) into Eq. (7), we have

$$\left\{\frac{\partial\Phi}{\partial n}\right\} = \left[\frac{\partial\Phi^*_T}{\partial n}\right][\Phi^*_T]^{-1}\{\Phi\}. \tag{9}$$

The stiffness matrix is obtained in a new way

$$[K] = \left[\frac{\partial\Phi^*_T}{\partial n}\right][\Phi^*_T]^{-1}. \tag{10}$$

Eq. (10) indicates that the strength of singularity is not required to be determined in advance to construct the stiffness matrix. For the same problem, there is the same stiffness matrix even though different methods are used. Therefore, we have

$$[B]^{-1}[A] = \left[\frac{\partial\Phi^*_T}{\partial n}\right][\Phi^*_T]^{-1} \Rightarrow [A]^{-1}[B] = [\Phi^*_T]\left[\frac{\partial\Phi^*_T}{\partial n}\right]^{-1}. \tag{11}$$

The result is the same to Eq. (6) of the paper [1]. We agree with Prof. Wu to point out that the intermediate process of derivation is not rigorous and easily misleads the readers. However, the final result of the paper by Zhang and Chen [1] is acceptable.

(2) *Occurrence of fictitious frequencies*: The authors in their paper claimed that the non-uniqueness problem appearing in the BEM is not present in the boundary point method. It may be not correct. It was demonstrated that irregular values also exist and shift to other positions for the method of fundamental solutions [8,9] and retracted BEM [10,11]. Since the boundary point method uses the fabricated source instead of concentrated source of MFS, it behaves like the retracted BEM [11]. It was theoretically proved and numerically demonstrated that the method of fundamental solutions [8] also encounter the irregular frequencies (non-uniqueness) as well as the boundary element method does [12]. The boundary point method and the method of fundamental solutions both are the meshless methods, which belong to the indirect BEM with concentrated source and fabricated sources distributing outside the domain, respectively. The main difference between the BPM and MFS is the singularity of a lumped source and a distributed fabricated source. In Refs. [8,9,11,13,14], the position of the irregular frequency depends on the location of source distribution and is different from that of direct BEM. The authors checked the irregular values of $k = 3.14$ and 6.28, 4.49 and 7.73 which are irregular values of direct BEM. This is not correct since the singularity is not located on the real boundary. It is expected that the non-uniqueness solutions were not found in the four locations

(3.14, 6.28, 4.49 and 7.73) in their paper. This cannot support the authors to claim that BPM is free of fictitious frequencies. The authors are encouraged to plot the response versus k value to see the new irregular values and can explain why irregular values shift. The authors can find figures from Refs. [8,14] for reference. Many researchers of MFS have investigated the study of irregular values. Until 2006, Chen [8] extended the circulant idea of Chen and his coworkers [9,12,14,15] to prove the existence of irregular values in MFS theoretically and to demonstrate numerically. This finding points out the wrong statement of "Why the non-uniqueness solutions were not found in this paper, the reason is that a discrete set of source points does not define an internal surface uniquely" as quoted from Fairweather et al. [16]. It is obvious that the boundary point method must encounter the non-uniqueness problem, too. There are several approaches to deal with the non-uniqueness problems, e.g., CHIEF [12] and CHEEF [17] methods, SVD updating techniques [9,10,13,15] and mixed-potential method [8,11,18–21].

References

[1] S.Y. Zhang, X.Z. Chen, The boundary point method for the calculation of exterior acoustic radiation problem, *Journal of Sound and Vibration* 228 (1999) 761–772.

[2] T.W. Wu, A.F. Seybert, A weighted residual formulation for the CHIEF method in acoustic, *Journal of the Acoustical Society of America* 90 (1991) 1608–1614.

[3] J.T. Chen, C.S. Wu, Y.T. Lee, K.H. Chen, On the equivalence of the Trefftz method and method of fundamental solutions for Laplace and biharmonic equations, *Computers & Mathematics with Applications* 53 (2007) 851–879.

[4] Sean F. Wu, Comments on "The boundary point method for the calculation of exterior acoustic radiation" (by S.Y. Zhang, X.Z. Chen, *Journal of Sound and Vibration* 228(4) (1999) 761–772), *Journal of Sound and Vibration*, 298 (2006) 1173.

[5] Sean F. Wu, Comments on "Reply to the comments on 'The boundary point method for the calculation of exterior acoustic radiation' (by S.Y. Zhang, X.Z. Chen, *Journal of Sound and Vibration* 228(4) (1999) 761–772)," *Journal of Sound and Vibration*, 298 (2006) 1176–1177.

[6] X.Z. Chen, C.X. Bi, Reply to the comments on "The boundary point method for the calculation of exterior acoustic radiation" (by S.Y. Zhang, X.Z. Chen, *Journal of Sound and Vibration* 228(4) (1999) 761–772), *Journal of Sound and Vibration*, 298 (2006) 1174–1175.

[7] X.Z. Chen, C.X. Bi, Reply to the comments on "Reply to the comments on 'The boundary point method for the calculation of exterior acoustic radiation' (by S.Y. Zhang, X.Z. Chen, *Journal of Sound and Vibration* 228(4) (1999) 761–772)," *Journal of Sound and Vibration*, 298 (2006) 1178–1179.

[8] I.L. Chen, Using the method of fundamental solutions in conjunction with the degenerate kernel in cylindrical acoustic problems, *Journal of Chinese Institute of Engineers* 29 (2006) 445–457.

[9] J.T. Chen, I.L. Chen, Y.T. Lee, Eigensolutions of multiply connected membranes using the method of fundamental solutions, *Engineering Analysis with Boundary Elements* 29 (2005) 166–174.

[10] J.T. Chen, L.W. Liu, H.-K. Hong, Spurious and true eigensolutions of Helmholtz BIEs and BEMs for a multiply connected problem, *Proceedings of The Royal Society of London Series A* 459 (2003) 1891–1924.

[11] J.Y. Hwang, S.C. Chang, A retracted boundary integral equation for exterior acoustic problem with unique solution for all wave numbers, *Journal of the Acoustical Society of America* 90 (1991) 1167–1180.

[12] I.L. Chen, J.T. Chen, M.T. Liang, Analytical study and numerical experiments for radiation and scattering problems using the CHIEF method, *Journal of Sound and Vibration* 248 (2003) 809–828.

[13] J.T. Chen, I.L. Chen, K.H. Chen, A unified formulation for the spurious and fictitious frequencies in acoustics using the singular value decomposition and Fredholm alternative theorem, *Journal of Computational Acoustics* 14 (2006) 157–183.

[14] J.T. Chen, K.H. Chen, I.L. Chen, L.W. Liu, A new concept of modal participation factor for numerical instability in the dual BEM for exterior acoustics, *Mechanics Research Communications* 26 (2003) 161–174.

[15] J.T. Chen, J.H. Lin, S.R. Kuo, S.W. Chyuan, Boundary element analysis for the Helmholtz eigenvalue problems with a multiply connected domain, *Proceedings of The Royal Society of London Series A* 457 (2001) 2521–2546.

[16] G. Fairweather, A. Karageorghis, P.A. Martin, The method of fundamental solutions for scattering and radiation problems, *Engineering Analysis with Boundary Elements* 27 (2003) 759–769.

[17] I.L. Chen, J.T. Chen, S.R. Kuo, M.T. Liang, A new method for true and spurious eigensolutions of arbitrary cavities using the CHEEF method, *Journal of the Acoustical Society of America* 109 (2001) 982–999.

[18] H. Brakhage, P. Werner, Uber das Dirichletsche Aussenraum-problem fur die Helmholtzsch Schwingungsgleichung, *Archiv der Math* 16 (1965) 325–329.

[19] R. Kussmaul, Ein numerisches verfahren zur Losung der Neumannschen Aussenraumproblems fur die Helmholtzsch Schwingungsgleichung, *Computing* 4 (1969) 246–273.

[20] R. Leis, Zur Dirichletschen randwertaufabe des Aussenraumes der Schwingungsgleichung, *Mathematische Zeitschrift* 90 (1965) 205–211.

[21] O.I. Panich, On the question of the solubility of the exterior boundary problem for the wave equation and Maxwell's equation, *Uspekhi. Mat. Nauk* 20 (1965) 221–226 (in Russian).